**Block Diagram Reduction Rules**

**DEFINITION:**
A block diagram of a system is a pictorial representation of the functions performed by each component of the system and shows the flow of signals. In block diagram, the system consists of so many components. These components are linked together to perform a particular function. Each component can be represented with the help of individual block.

**NEED FOR BLOCK DIAGRAM REDUCTION:**
Block diagrams of some of the systems turn out to be complex, such that the evaluation of their performance required simplification (or reduction) of block diagrams which is carried out by block diagram rearrangements.

**ADVANTAGES OF BLOCK DIAGRAM:**
- Very simple to construct the block diagram for complicated systems.
- Individual as well as overall performance of the system can be studied by using transfer functions shown in the block diagram.
- Overall closed loop transfer function can be easily calculated using block diagram rules.
- The function of the individual element can be visualized from the block diagram.

**DISADVANTAGES OF BLOCK DIAGRAM:**
- Block diagram does not include any information about the physical construction of the system.
Source of energy is generally not shown in the block diagram, so block diagram for a given system is not unique.

The basic components of block diagram are **block, branches, summing point, arrows**.

**Block:**

It indicates the function of particular system. \( R(s) \) is the reference or controlling variable. \( G(s) \) is the transfer function of the particular system. \( C(s) \) is output or controlled variable.

**Summing Point**

**Take Of Point**

**The steps to reduce the block diagram**

- Reduce the series blocks.
- Reduce the parallel blocks.
- Reduce minor feedback loops.
- As far as possible shift summing point to the left and take-off point to the right.
- Repeat the above steps till canonical form is obtained.

**Rules for reduction of block diagram**

**Rule 1:** If the blocks are in cascade then

\[ X \rightarrow G_1 \rightarrow G_2 \rightarrow Y \quad X \rightarrow G_1 G_2 \rightarrow Y \]

**Rule 2:** If the blocks are in parallel then, the blocks are added or subtracted depending on the summing point signal.

\[ \begin{align*}
R(S) \rightarrow & \quad G_1 \quad \rightarrow \quad G_2 \\
+ \quad & \quad + \\
\rightarrow & \quad C(s) \\
\end{align*} \quad = \quad \begin{align*}
\rightarrow & \quad G_1 + G_2 \\
\rightarrow & \quad C(S) \\
\end{align*} \]

**Rule 3:** Moving the take-off point after the block

\[ X \rightarrow \begin{array}{c}
G_1 \\
G_2 
\end{array} \quad \begin{array}{c}
\pm \\
\times 
\end{array} \quad Y \quad X \rightarrow G_1 \pm G_2 \rightarrow Y \]

**Rule 4:** Moving the take-off point after the block

\[ \begin{align*}
u \rightarrow & \quad G \\
u \quad & \quad y \\
u \rightarrow & \quad \frac{1}{G} \\
u \quad & \quad y 
\end{align*} \]
Rule 5: moving the take-off point before the block

Rule 6: Moving summing point after the block

Rule 7: Moving the summing point ahead (before) the block

Ex: Using block diagram reduction techniques find \( C(s) / R(s) \) as in figure below:
Step 1: eliminating feedback loop I

Step 2: eliminating feedback loop II

\[ G(s) = \frac{G_1}{1 + G_1H_1} \quad H(s) = H_2 \]

\[ \frac{C(s)}{R(s)} = \frac{\frac{G_1}{1 + G_1H_1}}{1 + (\frac{G_1}{1 + G_1H_1})(H_2)} \]

\[ 1 + G_1H_1 + G_1H_2 \]
Ex: Using block diagram reduction technique find closed loop transfer function \( C(s) / R(s) \) shown in figure below:

Step 1: Combine the blocks \( G_1 \) & \( G_2 \) which are in cascade and combine the blocks \( G_2 \) & \( G_3 \) which are in parallel.

Step 2: Eliminate feedback loop I and combine the blocks \( (G_1G_4 / 1 + G_1G_4H_1) \) & \( (G_2 + G_3) \) which are in parallel as shown.
Lec. 4

Step 3: eliminate feedback loop II

\[
\frac{C(s)}{R(s)} = \frac{\frac{G_1 G_4}{1 + G_1 G_4 H_1}}{1 + \frac{G_1 G_4}{1 + G_1 G_4 H_1}} \left( \frac{G_2 + G_3}{G_2 + G_3} \right)
\]

Ex: Determine the transfer function \( C(s) / R(s) \) of the system shown in Figure below by block diagram reduction method.
Step 1.

Shifting the summing point $S_2$ before the block $G_1$ and shifting the take off point $T_2$ after the block $G_4$.

Step 2.

Exchange the summing points and take off points using associative law and combining the series blocks we get.
Step: 3
Eliminating inner feedback loops I, II

Step: 4
Combine the blocks in series

Step 5:
Eliminate the feedback loop III

\[
C(s) = \frac{\frac{G_1G_2G_3G_4}{(1 + G_1G_2H_1)(1 + G_3G_4H_2)}}{1 + \frac{G_1G_2G_3G_4}{(1 + G_1G_2H_1)(1 + G_3G_4H_2)} \cdot \frac{H_3}{G_1G_4}}
\]

\[
C(s) = \frac{G_1G_2G_3G_4}{(1 + G_1G_2H_1)(1 + G_3G_4H_2) + G_2G_3H_3}
\]
H.W

1. Using block diagram reduction technique, find the transfer function from each input to the output $C(s)$ for the system shown in figure below:

2. Reduce the block diagram shown in fig.1.2.6.1. and obtain $C(s) / R(s)$
<table>
<thead>
<tr>
<th>Manipulation</th>
<th>Original Block Diagram</th>
<th>Equivalent Block Diagram</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Combining Blocks in Cascade</td>
<td>$X \rightarrow G_1 \rightarrow G_2 \rightarrow Y$</td>
<td>$X \rightarrow G_1 G_2 \rightarrow Y$</td>
<td>$Y = (G_1 G_2)X$</td>
</tr>
<tr>
<td>2 Combining Blocks in Parallel, or Eliminating a</td>
<td>$X \rightarrow G_1 \rightarrow \pm \rightarrow Y$</td>
<td>$X \rightarrow G_1 \pm G_2 \rightarrow Y$</td>
<td>$Y = (G_1 \pm G_2)X$</td>
</tr>
<tr>
<td>Forward Loop</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Moving a pickoff point behind a block</td>
<td>$u \rightarrow \bullet \rightarrow G \rightarrow y$</td>
<td>$u \rightarrow G \rightarrow y$</td>
<td>$y = Gu$</td>
</tr>
<tr>
<td></td>
<td>$u \rightarrow \bullet \rightarrow 1/G \rightarrow y$</td>
<td>$u \rightarrow \bullet \rightarrow \frac{1}{G} \rightarrow y$</td>
<td>$u = \frac{1}{G} y$</td>
</tr>
<tr>
<td>4 Moving a pickoff point ahead of a block</td>
<td>$u \rightarrow \bullet \rightarrow G \rightarrow y$</td>
<td>$u \rightarrow \bullet \rightarrow G \rightarrow y$</td>
<td>$y = Gu$</td>
</tr>
<tr>
<td>5 Moving a summing point behind a block</td>
<td>$u_1 \rightarrow \times \rightarrow G \rightarrow y$</td>
<td>$u_1 \rightarrow \bullet \rightarrow G \rightarrow y$</td>
<td>$e_2 = G(u_1 - u_2)$</td>
</tr>
<tr>
<td></td>
<td>$u_2 \rightarrow \times \rightarrow G \rightarrow y$</td>
<td>$u_2 \rightarrow \times \rightarrow G \rightarrow y$</td>
<td></td>
</tr>
<tr>
<td>6 Moving a summing point ahead of a block</td>
<td>$u_1 \rightarrow \times \rightarrow G \rightarrow y$</td>
<td>$u_1 \rightarrow \times \rightarrow G \rightarrow y$</td>
<td>$y = Gu_1 - u_2$</td>
</tr>
<tr>
<td></td>
<td>$u_2 \rightarrow \times \rightarrow G \rightarrow y$</td>
<td>$u_2 \rightarrow \times \rightarrow G \rightarrow y$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$u \rightarrow \bullet \rightarrow G_1 \rightarrow G_2 \rightarrow y$</td>
<td>$u \rightarrow \bullet \rightarrow \frac{1}{G_1} \rightarrow G_2 \rightarrow y$</td>
<td>$y = (G_1 - G_2)u$</td>
</tr>
</tbody>
</table>